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ABSTRACT

Various possibilities are discussed for hypothesis-generating and data analysis as they arise from the curriculum analysis component of the Second International Mathematics Study. The discussion covers three levels of investigation: (1) the curriculum as intended focuses on county- or system-level hypotheses; (2) the curriculum as implemented looks at hypothesized relationships between the system and classroom; and (3) the curriculum as learned discusses hypotheses linking system and student outcomes. (MP)

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THE CURRICULUM ANALYSIS: SOME HYPOTHESES AND IMPLICATIONS FOR THE SECOND INTERNATIONAL MATHEMATIC STUDY

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Paper prepared for the Annual Meeting of AERA
San Francisco, April 1979

SE 027 835

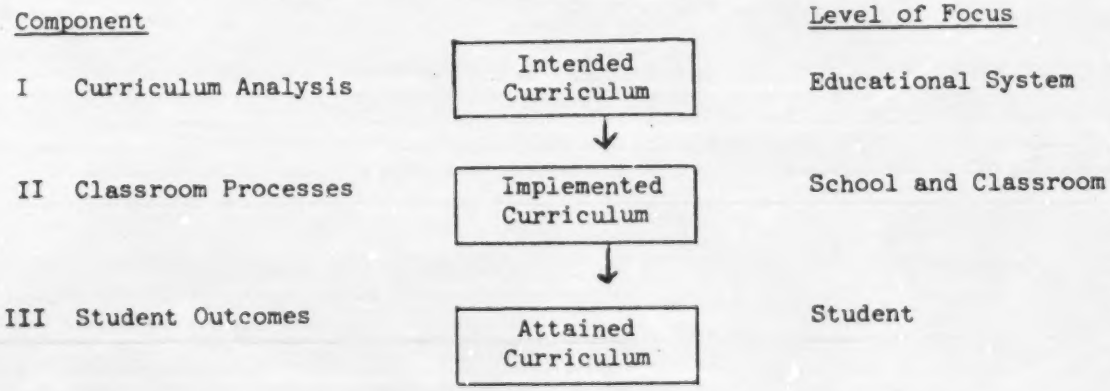
1. Introduction

The Second International Mathematics Study is an investigation of the teaching and learning of mathematics in schools. The Study has three main components: a curriculum analysis, which is an examination of the curricular contexts in which teaching and learning take place; a study of classroom processes, intended to provide information on what takes place in the classroom as mathematics is taught; and a survey of student outcomes (cognitive and affective) in the light of the nature of the curriculum and instructional practice. Two populations are targeted for study:

> Population A: All students in the grade enrolling the modal number of students which have attained the age of 13.0 - 13.11 by the middle of the school year.
>
> Population B: All students who are in the normally accepted terminal grade of the secondary educational system and who are studying mathematics as a substantial part (approximately five hours per week) of their academic program.

Figure 1 illustrates a conceptualization of the Study. Three levels are objects of investigation, one corresponding to each of the three components of the Study.

Figure 1. Conceptualization of the Study



1.1 First level: Educational system

The curriculum analysis focuses upon the educational system as an entity and is intended to portray the context in which mathematics education takes place (school organization, selectivity and educational goals, for example) as well as to review the status of mathematics education in that system.

The sources of data for the curriculum analysis include the following:

i. Royaumont follow-up questionnaire

In 1959, the Organization for European Economic Cooperation (OEEC) held a seminar in Royaumont, France, for the purpose of reviewing the current status of the school mathematics curriculum for the 21 member countries of OEEC and to consider the "production of a sound mathematics programme in harmony with modern thinking in mathematics." (New Thinking in School Mathematics. OEEC, 1961, page 7.)

The Curriculum Analysis Group* for the Second International Mathematics Study has prepared a questionnaire based on the Royaumont survey instrument, but modified in order to take account of mathematics topics which have since been "legitimized" in the curriculum and new topics, such as computer or information science which are candidates for "legitimacy", to varying degrees, in the curriculums of the IEA countries.

ii. Contexts of mathematics education

A questionnaire is being devised by the Curriculum Analysis Group in order to formulate a description of the societal and organizational contexts of mathematics education with a focus on occupational structure and national demography, articulation between secondary school and subsequent careers of students, school organization, curriculum development systems, control and coordinating systems (examinations, inspectorates, etc.) and the legal and credentialling structures of the system.

*The Curriculum Analysis Group, based at the University of Illinois, Urbana, consists of James Hirstein, Horacio Porta and Ian Westbury, of the University of Illinois at Urbana-Champaign, augmented at different times by Alan Purves of the University of Illinois at Urbana-Champaign; A. I. Weinzweig of the University of Illinois at Chicago Circle; Hans-Georg Steiner of the University of Bielefeld, FRG; Ulf Lundgren of the Stockholm Institute for Education, Sweden; and Roy W. Phillipps, Chairman of the IEA Mathematics Study Council, New Zealand. Kenneth Travers serves as liaison between the Curriculum Analysis Group and the International Mathematics Committee.

iii. National Case Studies

In 1977 a committee of the International Commission on Mathematics Instruction (ICMI) issued a call to various countries for reports on changes in mathematics education which have occurred since the late 1950's. The call included the following outline to be used as a guide to the authors of the national reports as they described the various developments in their countries:

Changes in:

- subject matter;
- teaching method;
- attitude toward mathematics;
- relations between mathematics and other subjects.

Change as a process:

The leading ideas --

- how did they develop in the course of the years?
- how did they manifest themselves in plans, projects, proposals?
- how were they actually realized?

The main forces --

- general educational, social, cultural developments;
- increasing mathematical literacy;
- increasing mathematical excellence;
- adult education;
- applicability of mathematics.

The designers --

- commissions, committees, curriculum developers, textbook writers, test designers.

The performers --

- administrations, schools, teachers, students, parents.

The tools --

- training, retraining, textbooks, syllabi, teachers' manuals.

Cooperation and resistance --

- by administrations, teachers, parents, public.

Lessons learned in the past to the benefit of the future.

Changes on different levels:

school, teacher training and retraining, and guidance in their inter-relatedness.

Reactions to change:

in other areas;
by higher learning;
by parents;
by the public.

Reports were received from the following countries (authors of the reports are given in parentheses) and published in Educational Studies in Mathematics, Volume 9, Numbers 2 and 3. Countries planning to participate in the Second IEA Mathematics Study are marked by *.

Australia* (A. L. Blakers)
Bangladesh (S. M. Sharfuddin)
France* (A. Revuz)
Great Britain (A. G. Howson)
Hungary* (Maria Halmos and Tamas Varga)
India (J. N. Kapur)
Iran (Badiollah Rostami)
The Netherlands* (H. Freudenthal)
Nigeria* (R. O. Ohuche)
Poland (A. Ehrenfeucht)
Sierra Leone (A. Williams)
Sri Lanka (A. J. Gunawardena)
Sudan (M. El Sawi)
Thailand* (O. Purakam)
United States* (J. T. Fey)
West Indies (B. J. Wilson)

Subsequently, reports from an additional half dozen countries were received and are to be published by UNESCO in Summer 1979.

Each IEA country will be invited to provide an update or other commentary on its national report as published in Educational Studies in Mathematics or the UNESCO document.

These reports will then be examined by a committee of three persons (Frederik van der Blij, the Netherlands; Sven Hilding, Sweden; and A. I. Weinzweig, the United States) which will prepare a summary paper synthesizing what is reported by the countries.

iv. Mathematics and mathematics education

The final component of the curriculum analysis will deal with three mathematical topics and relationships between the subject matter and how it is articulated in the curriculum and classroom. For illustrative purposes, three countries will be asked to prepare national case studies on the teaching of one of the three topics. The three topics are: geometry; introductory algebra; and probability and statistics.

1.2 Second Level: The Classroom

At this level, data will be sought in questionnaire form from both teachers and students.

i. Teacher coverage

The extent to which the curriculum as posed by ministry, school system, or some other authority outside the classroom is actually taught by the teacher has been a central concern of IEA studies since their inception in the early 1960's.

In this Study, coverage of the subject matter will be assessed by asking the teacher to rate each of the items on the cognitive test in response to this item:

> When is the first time that the majority of the students in your class were taught the subject matter on which this item is based?
>
> A. Prior to this school year
> B. During this school year
> C. Has <u>not</u> <u>yet</u> been taught this year(but will be)
> D. Taught in subsequent years
> E. Not in curriculum

ii. Instructional practice (this section applies only to Population A)

Information is sought on teacher behavior which is general (not expected to vary greatly across specific subject matter) and teacher behavior which is subject matter specific.

Sample items from the general teaching questionnaire are:*

PATTERNS OF CLASSROOM ORGANIZATION

	Never or Seldom	Occasionally	Frequently
a. All students in the class work on the same topics at the same pace.			
b. Students are grouped and each group works at its own pace.			
c. Each student learns and works at his/her own pace using his/her own individual material.			

USE OF INSTRUCTIONAL MATERIALS

	Never or Seldom	Occasionally	Frequently
a. Manipulatives (models, weights, sticks, rods, etc.)			
b. Games or puzzles			
c. Films or filmstrips			
d. Television or radio			
e. Computer facilities			
f. School library			
g. Community facilities (specify)			
h. Other (specify)			

*The general questionnaires have been developed under the direction of Graham Nuthall, Roy Phillipps and Jack Schwille. The topic specific questionnaires have been developed under the direction of Leslie Steffe and Thomas Cooney.

When you taught this topic to the target class, how much time (in minutes) did *you and the class* spend on each of the following activities?

A. *Lecturing:* Teacher explaining, talking, and lecturing to class or group of students with students listening.

Total Time _____ (minutes)

B. *Conference:* Explaining, talking to, and instructing individual students one or two at a time.

Total Time _____ (minutes)

C. *Remedial:* Providing additional *remedial* instruction to individual students with personal or mathematical difficulties.

Total Time _____ (minutes)

D. *Discussion:* Questioning, asking, and discussing with the whole class or group of students, with students participating by replying, commenting, reporting answers, etc.

Total Time _____ (minutes)

E. *Seat Work:* Students working on their own on written assignments, exercises, workbooks, or materials.

Total Time _____ (minutes)

F. *Large Group Work:* Students working together in several large (six students or more) groups talking, discussing, and helping each other on problems, assignments.

Total Time _____ (minutes)

G. *Small Group Work:* Students working together in small (two through five students) groups talking, discussing, and helping each other on problems, assignments.

Total Time _____ (minutes)

H. *Demonstration:* Teacher or student demonstrating audiovisual materials and models, showing films, etc. Students watching, touching, etc.

Total Time _____ (minutes)

I. *Copying:* Teacher reading or writing on blackboard, showing projected image, with students copying, writing in their books.

Total Time _____ (minutes)

J. *Management:* Teacher or students arranging furniture, preparing audiovisual equipment, moving to new groups, etc.

Total Time _____ (minutes)

Sample items from the topic specific questionnaire are:

THE PYTHAGOREAN THEOREM

1. Indicate which of the following are techniques that you use in teaching the Pythagorean Theorem.

   a. I present my students with a variety of right triangles and have them measure and record the lengths of the legs and hypotensure. The pattern is discussed and then we state the theorem. ..... Yes No

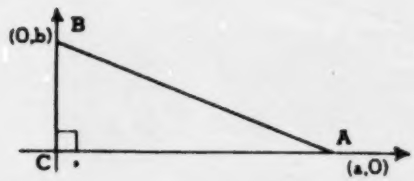
Example:

leg	leg	hypotenuse
3	4	5
9	12	15

$3^2 + 4^2 = 5^2$

$9^2 + 12^2 = 15^2$

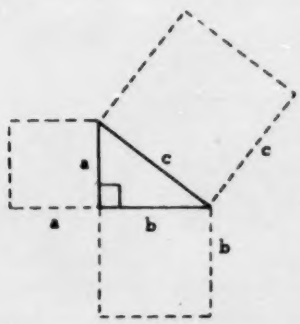
   b. Using Cartesian coordinates, we develop the Pythagorean rule by placing a right triangle in standard position and then use the formula for the distance between two points to find the length of the hypotenuse. ..... Yes No



$\overline{AB} = \sqrt{(a-0)^2 + (b-0)^2}$

$\overline{AB} = \sqrt{a^2 + b^2}$

   c. I use diagrams like the following to show that $c^2 = a^2 + b^2$: Yes No



   d. I give my students the formula and have them use it in working examples. Yes No

   e. I use the geoboard to establish the relation between the hypotenuse and the two legs of any right triangle. ..... Yes No

   f. I emphasize applications of the Pythagorean rule to real-life situations. Yes No

1.3 Third Level: The Student

Measures on student achievement and attitudes will be obtained as described in Bulletin 3. The cognitive tests are described in pages 28-30 and the attitude measures are described in pages 30-31.

2. Country or System Level Hypotheses

2.1 The Status of Mathematics Education

The time frame which is being considered for the curriculum analysis - the early 1960's until the present - is a useful one for several reasons. It has already been pointed out that the Royaumont survey (held in 1959) provides information on the subject matter composition of the curriculum at that time, the period just prior to extensive curriculum reform activity in most countries. The national reports in Educational Studies in Mathematics were written to cover roughly this same period of time. It is also fortunate, of course, that the first IEA mathematics study took place in 1964. Thus, this period of almost 20 years may represent a complete cycle. That is, in many countries it may well be the case that there is now "retrenchment" in the curriculum. The "new mathematics" may be viewed with suspicion, if not alarm. There may be questions as to the long term benefit of the curriculum reform movement, in terms of attainment of arithmetic skills and concepts, problem solving, and preparation for college level work in the sciences and engineering as well as mathematics. The present time affords an unusually propitious opportunity to survey the status of the curriculum and to examine what changes have actually occurred. But because of the possibility of retrenchment, it is important to consider not only the endpoints of the period, but to trace developments and activities over the entire period. This is best done, perhaps, by narrative and exposition rather than by narrowly focused questionnaires.

Whither Euclid?

One of the most dramatic and highly publicized positions taken at the Royaumont conference, for example, was that of the French mathematician and member of the exclusive Bourbaki Group, Jean Dieudonné. The major theme of the entire Royaumont conference was, to be sure, "the production of a sound mathematics programme in harmony with modern thinking in mathematics" (page 7, The Report). Dieudonné pushed extremely hard on this point, however, lamenting

the tendency of schools to resist new terminology, to desperately cling to obsolete and inadequate language. In his words,

> .....when a student enters the university, he will most probably never have heard such common mathematical words as set, mapping, group, vector space, etc. No wonder he is baffled and discouraged by his contact with higher mathematics.
>
> Some elements of calculus, vector algebra and a little analytic geometry have recently been introduced for the last two or three years of secondary school. But such topics have always been relegated to a subordinate position, the centre of interest remaining as before "pure geometry taught more or less according to Euclid, with a little algebra and number theory".
>
> I think the day of such patchwork is over, and we are now committed to a much deeper reform--unless we are willing to let the situation deteriorate to the point where it will seriously impede further scientific progress. And if the whole program I have in mind had to be summarized in one slogan, it would be: Euclid must go! (pages 34-35, The Report)

Now, twenty years later, it is important to know the extent to which the reforms proposed by Dieudonne and others have had an impact on mathematics in schools. Dieudonne, for example, proposed a curriculum "at last freed from the deadweight of 'pure geometry'", suggesting in its place preparatory work in such areas as matrices and determinants of order 2 and 3; elementary calculus (functions of one variable); construction of the graph of a function and of a curve given in parametric form (using derivatives); elementary properties of complex numbers and polar coordinates (page 38, The Report)

The use of the two survey instruments, the original Royaumont questionnaire and its revised form as prepared by the Curriculum Analysis Group should provide data upon which to answer this question. It would seem that the response will encompass a variety of answers depending upon the extent to which a country has been influenced by the curriculum reform movement. We would expect to find some countries which have undergone a drastic reform, a true revolution, in the school mathematics curriculum. We would expect to find other countries in which, for a variety of reasons, little substantive change has occurred. There will also be a third group which is still dealing with the issues and controversies raised during the reform movement. First, they may have been moved one way, then another, and at present may not have established clear positions or goals concerning these curricular issues.

We would expect that for all countries, the information provided not only by the curriculum analysis phase of the Study, but by the survey of instructional practice and student outcomes, would be of great value as decisions are made concerning curricular goals for the 1980's.

2.2 The character of school mathematics

As the Curriculum Analysis Group states it, educational systems are periodically subjected to impulses. These may be economic, as in periods of inflation or depression; they may be political, as when surges of nationalism reject foreign input; or these impulses may be disciplinary or subject-matter oriented, as was clearly the case during the curriculum reform movement of the 1960's.

One probe of the curriculum analysis is designed to explore the effect which impulses from the field of mathematics have had upon curriculums in various countries. Three topics are chosen: geometry, elementary algebra and probability and statistics. For each of these topics, three countries will be invited to develop case studies which trace the impulses which have impinged upon their curriculums and the effects which these forces have had upon what and how mathematics is taught in their schools.

i. Geometry

In view of the controversy which surrounded this topic at the Royaumont conference, and the considerable activity in curriculum development which has occurred in subsequent years, this topic is a clear candidate for selection.

ii. Elementary algebra

It is probably accurate to state that the curriculum reform movement began in earnest with elementary concepts in algebra. In the 1950's, Max Beberman and his group at the University of Illinois had been exploring the concept of a variable with his frames and "chalkboard with a hole in it" for representing open sentences. Sets, used to express notions of solutions of equations and inequations and to motivate mathematical operations such as union and intersection at early stages of the school curriculum, took on such prominence that the term seemed to become synonymous with "new mathematics".

Case studies from selected countries should be useful in illustrating how such ideas are introduced into the curriculum (or why they are rejected) and the embodiments such impulses assume within the system.

iii. Probability and statistics

This topic also received attention at Royaumont. As part of the summary and conclusions of that meeting it was stated, under the heading, "Probability and Statistical Inference":

> These topics, particularly the second, are recommended as a desirable new field of instruction in the secondary school. While probability, based on a priori consideration of the number of possible combinations (and permutations), has sometimes been a part of secondary-school algebra, the use of a posteriori considerations from experiments and the use of confidence limits is new (page 117, The Report).

Our initial findings, based upon preliminary reports from countries on subject matter emphases for the 13 year olds and the terminal students, strongly suggest that relatively little has taken place in this area, twenty years later. However, there are pockets of activity (Sweden and England, for example) and we wish to trace the sources of the impulses bringing about this activity.

The case of statistics merits special attention because its roots are outside of the field of mathematics, notably, agriculture and psychology, and brings with it an applied tradition and "real world problems". The interface of such a topic with the more "orthodox" topics of geometry and algebra, and the associated emphases and instructional method which emerge in the curriculum should provide instructive examples of how the character of school mathematics is shaped.

2.3 Mathematics for the Majority

Another issue was raised at Royaumont, which in many ways seemed to have been in conflict with the press from mathematicians for a "modern" curriculum, was that of providing a sound mathematical education for all students (even though Dieudonné himself seemed to go out of his way to assure the meeting that he was concerned not with the training of future mathematicians but with "the kind of mental picture of mathematics which will emerge in the mind of the average intelligent student after he has been subjected to that treatment for several years").

Marshall Stone, in his introductory address to the conference, remarked that "the teaching of mathematics (is) coming to be more and more clearly recognized as the true foundation of the technological society which it is the destiny of our times to create" and went on to recognize "the task of organizing mass education at the secondary level is made more difficult ... by the necessity of dealing effectively with large numbers of students widely spread out on the scale of intelligence and similarly scattered in respect to motivation and interests" (page 18, The Report).

At this juncture, as we go into the 1980's, it appears that little progress has been made toward this goal of mass education at the secondary level, or as it has been described by some, "mathematics for the majority". The dual problems of what mathematics should be learned in order for individuals to be functional, effective citizens in a modern, technological society, and what pedagogical methods should be employed to bring about this desired knowledge could well delineate dominating areas of research and development in mathematics education in the declining years of the twentieth century. Baseline data at this point will be of great value as progress in subsequent years of investigation is assessed.

3. Hypothesized Relationships between the System and Classroom

One aspect of the linkage between the curriculum at the system or national level and the classroom is that of teacher coverage or "opportunity-to-learn". One view of such a measure is to regard it as an index of the curricular validity of the international tests (the greater the extent to which the subject matter has actually been taught in a country's classrooms, the more appropriate is that test as a measure of student achievement in that country and as a corollary, the more meaningful are cross-national comparisons on that test.) As in past IEA studies, we would expect that teacher coverage will be a relatively powerful variable in terms of accounting for variance in student achievement.

There is another direction in which interpretation of the teacher coverage index could move, and about which I am uncomfortable. That is, one could speak of congruence between the intended and implemented curriculum

as some sort of measure of teacher competence or effectiveness, and contemplate some sort of ranking of national systems of education on the basis of the degree to which the systems' teachers implement the national curriculum.

Such an interpretation is fraught with pitfalls, however. A fundamental problem is that teaching is not some mechanistic process whose success depends upon the degree to which desired information can be delivered, or "laid on" students. The essential dynamics of a classroom, the sensitive interaction between teacher and student, the judicious selection of appropriate subject matter and instructional approach for a given day are among the factors which are to be taken into account in an educational system. Teacher coverage is an important measure, but is only one of a vast complex of critical variables which enter into the mix for effective teaching and learning.

For the remainder of this section, I will confine my remarks to three subject-matter oriented probes into the curriculum: they are centered upon geometry, introductory algebra, and statistics. The fundamental notion to be explored in each of these probes is the linkage between how the subject matter is viewed and how it is taught.

3.1 Geometry

This investigation has already been alluded to in the discussion of the Royaumont seminar in 1959. Let us say, for example, that three countries, Alpha, Beta and Gamma have embarked upon rather different courses in terms of curricular thrusts. Alpha, say, has responded to the admonition that "Euclid must go" and has set about to work out the implications of that dictum by utilizing a "unified" approach to mathematics involving a Vector approach. Gamma, on the other hand, has remained rather conservative in its approach to geometry in that much of conventional Euclidean geometry remains in its curriculum. There is an important development, however, in that consonant with the country's traditional orientation to a more applied, practical view of mathematics, there has been a great deal of activity in the area of transformational geometry. Hence, while most of the theorems from the conventional course remain, they are approached from a more active, intuitive point of view.

Beta, meanwhile, has been tossed to and fro in the debate over geometry. Emmisaries representing varying points of view have come to its shores many times during the past fifteen or so years. And while in some parts of the country, there have been experimental programs of various sorts, there is no clear cut curriculum in geometry. "It all depends ... " This lack of clarity is reflected in teacher responses and in student performance.

One might expect differences between curricular emphases and mathematical tradition in Alpha, Beta, and Gamma to occur along dimensions of classroom process which are being explored in current drafts of the Geometry classroom process questionnaire.

i. Emphasis

Indicate (check) the degree of emphasis that each of the following approaches to geometry received in your instructional program.

	Heavy Emphasis	Moderate Emphasis	No Emphasis
a. An informal transformational approach (translations, rotations, reflections, etc.)	GAMMA		
b. An informal measurement approach (lengths, areas, volumes, perimeters, etc.)		BETA	
c. An informal non-measurement approach (Properties of planar figures, classification of quadrilaterals, etc.)		BETA	
d. A deductive Euclidean approach (Proof of theorems, axioms, definitions, etc.)		BETA GAMMA	ALPHA
e. A deductive transformatio nal approach		ALPHA	
f. A coordinate approach (Coordinates of points, graphing of equations, etc.)		ALPHA	
g. A vector approach (addition of ordered pairs, a scalar times an ordered pair, distance formula, etc.)	ALPHA		

ii. Teaching techniques (sum of measures of angles in a triangle)

Various alternatives for teaching the concept that the sum of the measures of the angles of a triangle is equal to 180 degrees are provided in the questions as shown in pages 14, 15 and 16. The hypothetical result for our three countries sugests a more more formal, deductive approach for Alpha as compared with an experience-based, intuitive approach for Gamma.

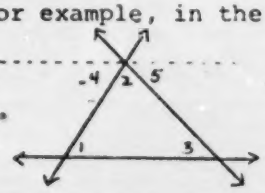
For each technique, check the response that best describes your use of it:
N or S (Never or Seldom; O (Occasionally); F (Frequently).

a. My students measured the angles of a triangle and added the measures to discover that the sum of the measures is 180°.

N or S	O	F

b. I drew a line through a vertex parallel to the opposite side and used alternate interior angles to show that the sum of the angles of a triangle is 180°.

For example, in the figure $\measuredangle 1 = \measuredangle 4$ and $\measuredangle 3 = \measuredangle 5$,

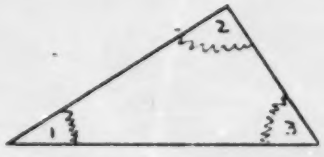


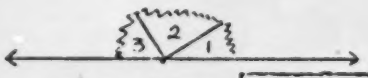
So $\measuredangle 1 + \measuredangle 2 + \measuredangle 3 = \measuredangle 4 + \measuredangle 2 + \measuredangle 5$

$= 180^{\circ}$

N or S	O	F

c. My students cut the angles off a triangle and arranged them on a straight line.



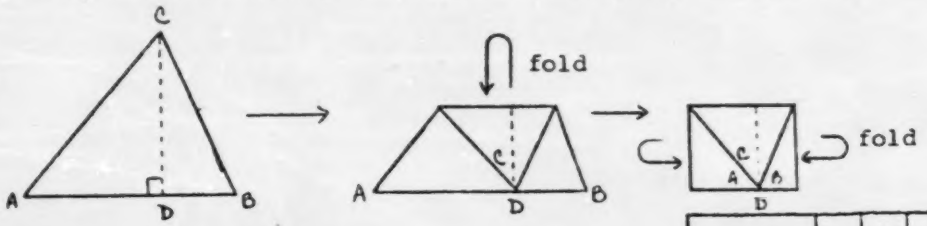


N or S	O	F

d. I told my students that the sum of the measures of the angles of a right triangle is 180° and had them verify it by measuring the angles and adding the measures.

N or S | O | F

e. I had my students verify the relationship by paper folding.

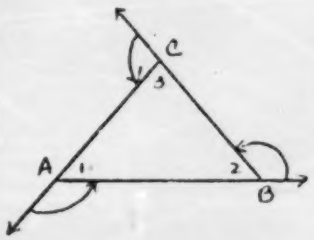


N or S | O | F

f. I inscribed a triangle in a circle and used the facts that a circle contains 360° and inscribed angles are 1/2 of the corresponding central angle.

N or S | O | F

g. I used the fact that (as Illustrated in the figure) in traveling $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$, a complete revolution (360°) is swept.

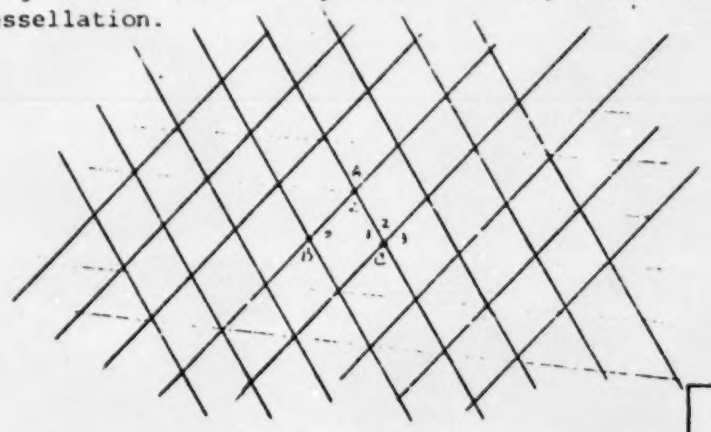


Using this and angle supplements,

$\measuredangle 1 + \measuredangle 2 + \measuredangle 3 = 180^{\circ}$

N or S | O | F

h. Using tessellations, I identified three angles at a point (C) congruent with three angles in a triangle (ABC) embedded in the tessellation.



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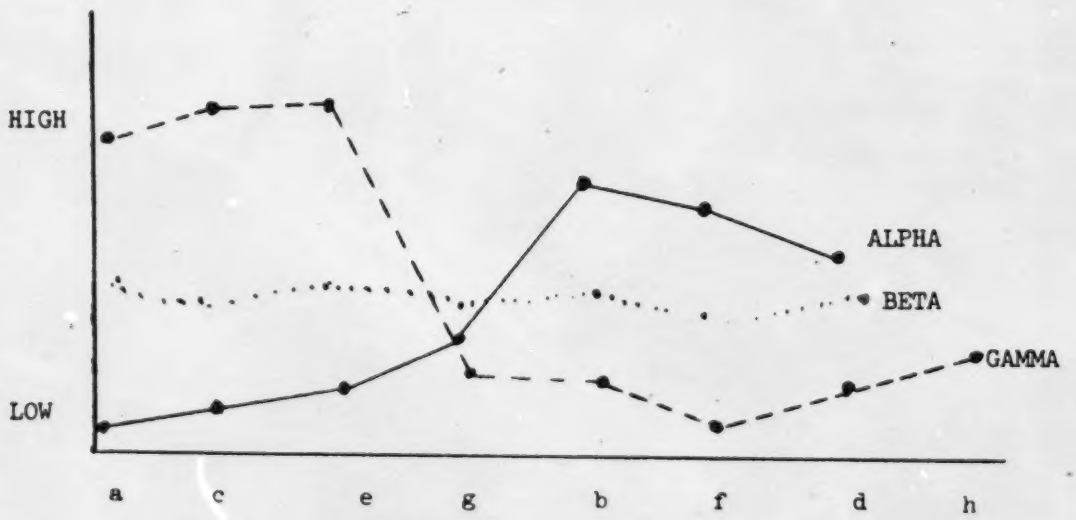


Techniques Used to Teach
Sum of Measures of Angles of a Triangle

iii. Justification strategies

A companion item to the previous one is designed to reflect various approaches to justifying generalizations in geometry.

	Never or Seldom	Occasionally	Frequently
a. Teacher authority: I tell my students that the generalization is true and the students use it.		ALPHA	
b. Justification by example: I present enough examples to convince my students that the generalization holds.		BETA	GAMMA
c. Deductive argument: I follow logical reasoning based on definitions, axioms and previous theorems.		BETA	ALPHA
d. Plausible reasoning: I call upon the plausibility of the generalization.		ALPHA GAMMA	

4. Hypotheses Linking System and Student Outcomes

This class of comparisons investigates possible relationships between charteristics of the curriculum and accompanying student outcomes.

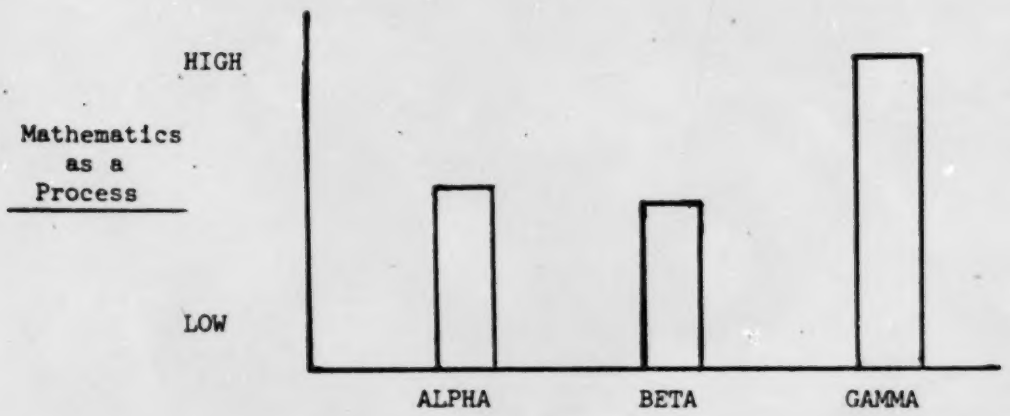
4.1 Attitudinal Dimensions

There is considerable overlap between the attitude scales of the first and second mathematics study. This should permit useful benchmark comparisons for countries which participated in the first study. In particular, countries in which student attitudes were manifestly negative in the first study may wish to identify possible shifts on these measures. The Mathematics as a process scale, designed to measure one's view of the nature of mathematics may also be examined in the light of curricular emphasis in the various countries. One might hypothesize, for example, that curricular emphases in Alpha and Gamma, combined with the above projected profiles of classroom process in those countries, would be accompanied by Mathematics as a Process outcomes such as suggested below.

Mathematics as a Process

This scale is intended to obtain a measure of the student's perceptions about the nature of mathematics. It is hypothesized to reflect differences in curriculums and in classroom processes. For example, emphases on mathematics as a creative enterprise, and classroom approaches that encourage open-ended investigations on the part of students, are expected to impart a view of mathematics that is dynamic and creative.

Figure 2.



4.2 Cognitive dimensions

The most popularized aspects of this sort of study are, undoubtedly, the student outcome measures, and particularly the cognitive aspects. In view of the considerable curricular activity over the past 20 years, one should expect attention to focus upon comparisons between student achievement "then and now". The results of the National Longitudinal Study of Mathematical Abilities, which examined student outcomes over a five year period beginning in 1962 for curriculums based on "conventional" and "modern" mathematics textbooks, suggest the sort of findings one might expect for the United States, and, assuming reasonably comparable definitions of "conventional" and "modern", and accompanying implementation in the classrooms (bold assumptions, to be sure!) one might expect that countries which have undergone extensive "modernizing" of the curriculum will exhibit student outcome measures which have increased on the upper levels of cognitive behavior while at the same time some loss of ground on the lower levels might be anticipated. On the other hand, countries which have experienced less "modernizing" of the curriculum might expect less shifting across behavioral levels and (again, other factors being equal, which they never are) more congruence of student outcomes between the first and second studies.

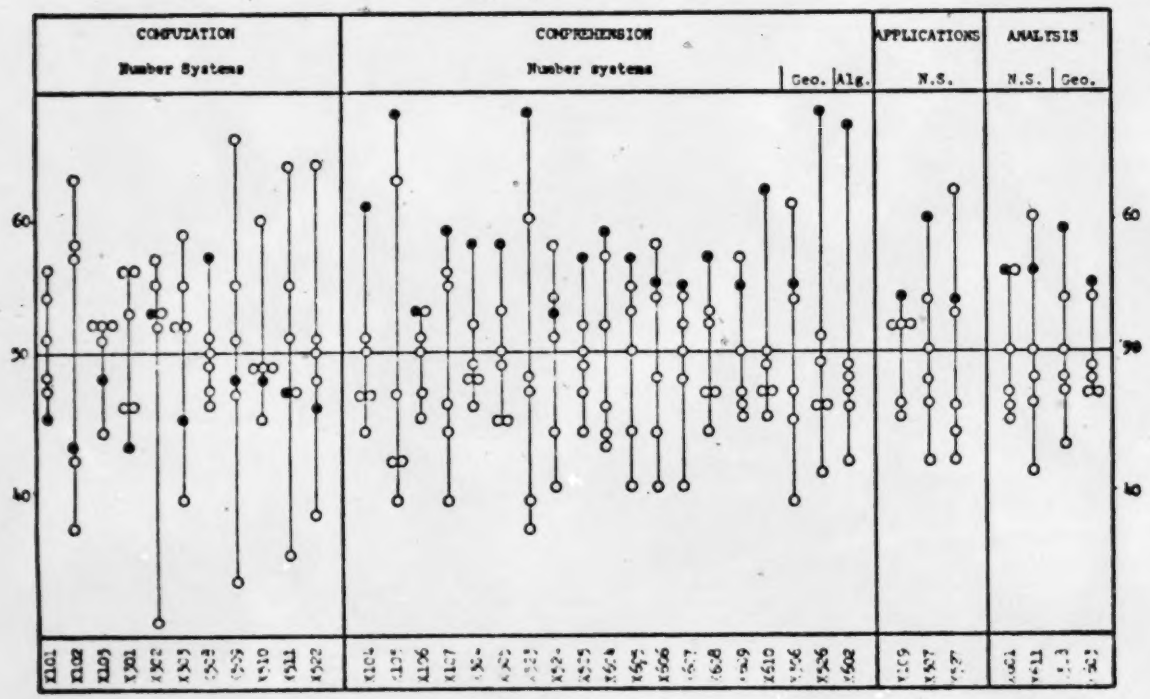


Fig. 3 - Profile for Textbook Group T1 (X-Population; Grades 4, 5, 6; Years 1, 2, 3)

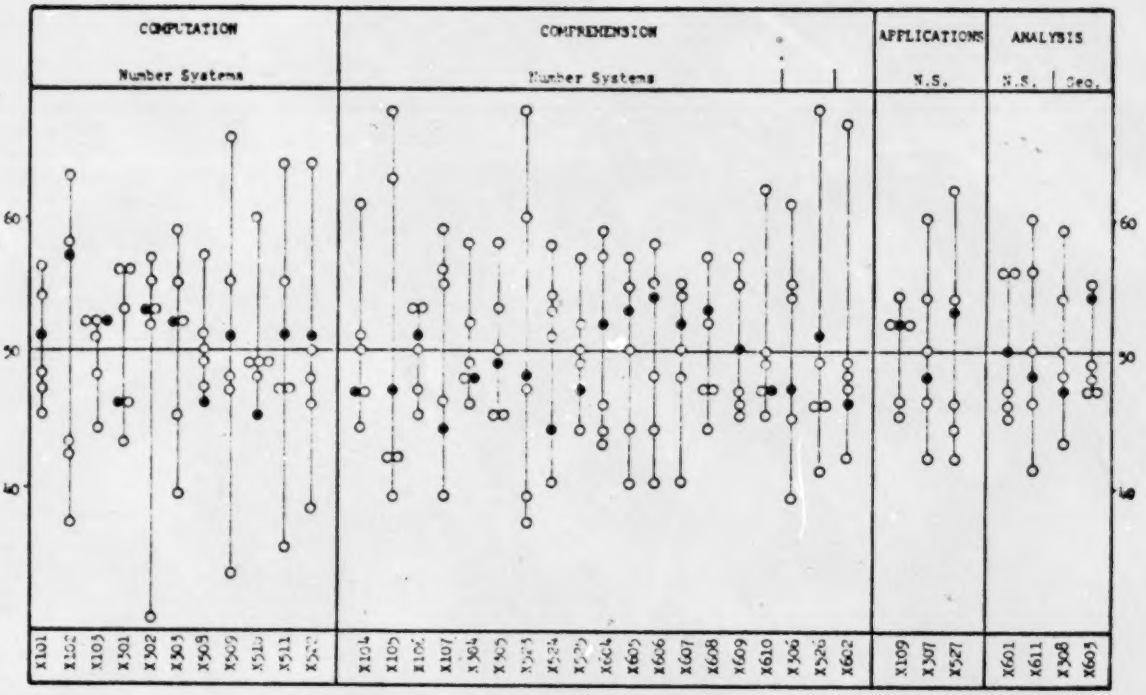


FIG. 4.—Profile for Textbook T1 (X-Population; Grades 4, 5, 6; Years 1, 2, 3)

(The preceeding two figures are from E. G. Begle and J. W. Wilson, "Evaluation of mathematics programs," Mathematics Education, Sixty-ninth Yearbook of the National Society for the Study of Education, 1970, pages 396 and 398, respectively.)

The figures present mean achievement scores for students in "modern" (Textbook T_1) and "traditional" (Textbook T_2) textbook groups, transformed to standardized scores having a mean of 50 and standard deviation of 10. The circles in each figure represent textbook groups. Therefore, the figures present profiles illustrating the relative positions of the various groups on 37 cognitive scales used in the survey. In Figure 3 the darkened dot represents a "modern" text, the SMSG program. Notice the relative position of these groups on the lower cognitive level as to the higher levels. In the second figure (Figure 4) the darkened dot

represents a conventional text. The relative position of this group is in many respects a striking contrast to that of the modern text.

Item Level Information

As may already be evident in this paper, and has been explicity stated elsewhere (see, for example, Bulletin 3, pages 36-37), the International Mathematics Committee wishes to focus upon simple description of outcomes at the subscore or item level in an attempt to present findings which are most easily understood and communicated. There is also the intention, however, to preserve the data in a format which enables analyses based upon a variety of models.

One of the potentially most valuable sources of information out of the first mathematics study, which still remains to be fully mined and is reported in Appendix II to the report of that study, is the item level information, indicating those countries in which each item was hardest and in which it was easiest. A sample from the Population 1 (our Population A) results is provided.

	Target Population	Average Difficulty	Average Discrimination	Order of Error Frequency	Easier in	Harder in
7. A box has a volume of 100 cc. Another box is twice as long, twice as wide and twice as high. How many cc is the volume of the second box? *Ans. 800*	1a	.21	.39		—	Australia, Sweden, United States
	1b	.24	.40		Belgium, Israel	Australia, Sweden, United States
8. On the scale to the right, the reading indicated by the arrow is between A. 51 and 52 B. 57 and 58 C. 60 and 62 D. 62 and 64 *E. 64 and 66	1a	.42	.45	B, D	England, Japan	Belgium, France
	1b	.44	.44	B, D	England, Japan	Belgium, France, Israel

One can propose many hypotheses for explaining the differential difficulty of these items in various countries. If it can be assumed that Item 8 has more of a "real world" flavor than Item 7, what is it about the curriculum in England and Japan which might account for the relative ease of the scale reading task? Is the applied nature of mathematics much more evident in the curriculum of those countries than in Belgium and France? Item 7, on the other hand, has its own intriguing qualities. Does success on this item reflect understanding of volume in any deep sense or the ability to use formulas? And again, what in the curriculum might account for the differences?

Certainly, instructional effects must be taken into account as well. The kinds of experiences in which children are engaged as learning takes place would surely be reflected in performance on items such as these. Children who have actually stacked blocks and counted as they learned the concept of volume could be expected, generally, to be more successful at solving problems like the one posed in Item 7. But, on the other hand, prior experience in applying mathematical formulas might overrule and account for the success of, say, the Belgians.

For a more detailed and imaginative treatment of this topic, as well as others addressed in this paper, the reader is urged to read the paper by Kifer presented at the Michigan State Symposium in February 1979.

SUMMARY

This rather wide-ranging discussion has touched on various possibilities for hypothesis generating and data analysis as they arise from the curriculum analysis component of the Study. The fundamental point, however, that has been made throughout the planning and development phases of the Study, is that there is an overriding concern for probing beneath the surface of the student outcome data which will be obtained. We expect that considerable variation in these outcomes will be found across countries. This variation is due to many causes, to be sure. But two sources must be the curriculum and classroom practice. Getting some sort of a handle on these, however crude the measures may be, represents an important step forward in understanding what is important in the mix of teaching and learning in the mathematics' curriculum.

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